Solutions

April 25, 2025

Math 1A Worksheet #33

Name: ______________________________

$\cos(2x) = 2\cos^2(x) - 1$
$\Rightarrow \int \cos^2(x)\,dx = \int \frac{1}{2}(1+\cos(2x))\,dx$

1. Verify the following formulae.

(a) $\displaystyle\int \cos^2 x \, dx = \frac{1}{2}x + \frac{1}{4}\sin(2x) + C$

(b) $\displaystyle\int x\sqrt{a+bx} \, dx = \frac{2}{15b^2}(3bx - 2a)(a+bx)^{\frac{3}{2}} + C$

$\int x\sqrt{a+bx}\,dx = \int \left((a+bx)^{3/2} - a\sqrt{a+bx}\right)\frac{1}{b}\,dx$
$\frac{2}{5}\left(\frac{1}{b^2}\right)(a+bx)^{5/2} - \frac{a}{b^2}\left(\frac{2}{3}\right)(a+bx)^{3/2} + C$

2. Compute the following indefinite integrals.

(a) $\displaystyle\int \sqrt[4]{x^5} \, dx$ $= \int x^{5/4}\,dx = \frac{4}{9}x^{9/4} + C$

(b) $\displaystyle\int \left(u^6 - 2u^5 - u^3 + \frac{2}{7}\right) du$ $= \frac{1}{7}u^7 - \frac{1}{3}u^6 - \frac{1}{4}u^4 + \frac{2}{7}u + C$

(c) $\displaystyle\int \left(x^2 + 1 + \frac{1}{x^2+1}\right) dx$ $= \frac{1}{3}x^3 + x + \arctan(x) + C$

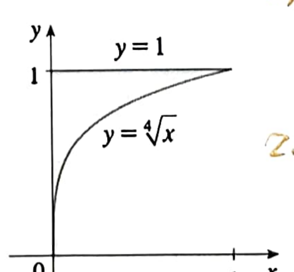
(d) $\displaystyle\int \frac{\sin(2x)}{\sin x} \, dx$ $= 2\sin(x) + C$

(e) $\displaystyle\int (5x - 5^x) \, dx$ $= \frac{5}{2}x^2 - \frac{5^x}{\ln(5)} + C$

3. Let $f(x)$ be the slope of a hiking trail at a distance of x miles from the trailhead. What does $\int_3^5 f(x)\,dx$ represent? Change in elevation from 3->5 miles.

4. Find the area of the region enclosed by the y-axis, the line $y = 1$, and the curve $y = \sqrt[4]{x}$. Hint: there are 2 ways to do this!

1.) $\int_0^1 y^4\,dy = \frac{1}{5}$



2.) $1 - \int_0^1 x^{1/4}\,dx = 1 - \frac{4}{5} = \frac{1}{5}$

5. The acceleration of a particle traveling along a line as a function of time t is given by $a(t) = 2t + 3$. The velocity of the particle at time $t = 0$ is $v(0) = -4$. Find (a) $v(t)$, the velocity of the particle at time t, and (b) the displacement of the particle during the time $0 \le t \le 3$.

A.) $v(t) = t^2 + 3t - 4$ B.) $\frac{1}{3}(3)^3 + \frac{3}{2}(3)^2 - 4(3)$

6. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, for $0 \le t \le 50$. Find the amount of water that flows from the tank during the first 10 minutes.

2000 − 200 = 1800 Liters.

7. Using the fundamental theorem of calculus, simplify

$$\frac{d}{dx}\int_{\sin x}^{1+\cos x} \sqrt{1+t^3}\, dt.$$

U-Sub

- Process Goal: Formalize a way to "undo the chain rule"

General form:

$$\int f(g(x))g'(x)dx$$

Set $u = g(x)$

$du = g'(x)dx$

$$\int f(\underbrace{g(x)}_{u})\underbrace{g'(x)dx}_{du} = \int f(u)du$$

e.g. $\int \frac{2x}{3+x^2} dx$ $\quad u = 3+x^2$, $\quad du = 2x\,dx$

$$= \int \frac{1}{u} du = \ln|u| + C = \ln|3+x^2| + C$$

$\int_2^5 \frac{\ln(x)}{x} dx$ $\quad u = \ln(x)$, $\quad du = \frac{1}{x} dx$

$$\int_{x=2}^{x=5} u\,du = \frac{1}{2}u^2\Big|_{x=2}^{x=5} = \frac{1}{2}\ln(x)^2\Big|_2^5$$

Sol

Math 1A Spring 2025 Quiz 8

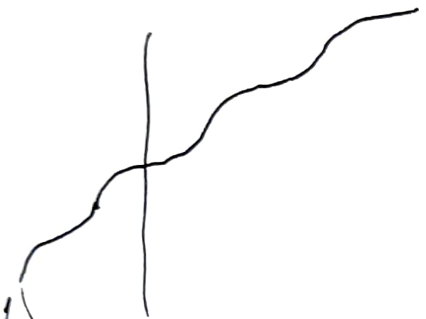
Name:

1. Sketch the curve $y = x + \cos x$.

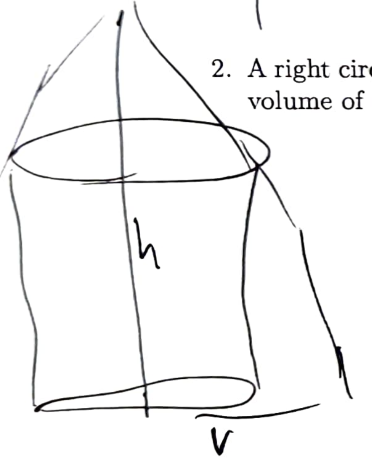
$y' = 1 - \sin(x)$, Crit. pt.s $\rightsquigarrow \pi/2 + 2k\pi$; Never max/min's

$y'' = -\cos(x)$ C.U. on $(\pi/2 + 2\pi k, 3\pi/2 + 2\pi k)$

C.D. $(3\pi/2 + 2\pi k, \pi/2 + 2\pi(k) + \pi)$



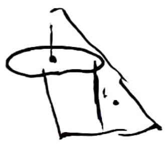
2. A right circular cylinder is inscribed in a cone with height h and base radius r. Find the largest possible volume of such a cylinder.



$V = \pi r_2^2 h_2$

$\frac{h_2}{r - r_2} = \frac{h}{r} \to \cancel{h(r - r_2) = r h_2}$

$h_2 = \frac{h}{r}(r - r_2)$



$V = \pi r_2^2 \left(\frac{h}{r}\right)(r - r_2)$

$V' = \pi\left(\frac{h}{r}\right)(2 r_2 r - 3 r_2^2)$

$= \pi\left(\frac{h}{r}\right) r_2 (2r - 3r_2)$

Crit. pt.: $r_2 = 0$

$r_2 = \frac{2}{3} r$

$\cancel{V = \pi\left(\frac{2r}{3}\right)\left(\frac{h}{r}\right)\left(r - \sqrt{\frac{2r}{3}}\right) = \frac{2\pi h}{3}\left(r - \sqrt{\frac{2r}{3}}\right)}$

$V = \frac{4}{27} \pi r^2 h$

3. Find f given $f''(x) = \sqrt[3]{x} - \cos x$ and $f(0) = 2$ and $f(1) = 2$.

$f' = \frac{3}{4} x^{4/3} - \sin(x) + C_1$

$f = \left(\frac{3}{4}\right)\left(\frac{3}{7}\right) x^{7/3} + \cos(x) + C_1 x + C_2$

$f(0) = 1 + C_2 = 1 \Rightarrow C_2 = 1$

$f(1) = \frac{9}{28} + \cos(1) + C_1 + 1 = 2$

$C_1 = 2 - 9/28 - 1 - \cos(1)$

$19/28 - \cos(1)$

$f(x) = \frac{9}{28} x^{7/3} + \cos(x) + \left(\frac{19}{28} - \cos(1)\right)x + 1$